Supplement 6. HWR Measure Statistical Approach to Risk-Standardized Readmission Rates

We estimate the hospital-specific RSRR using mixed effects logistic regression models with hospital random effect, a strategy that accounts for within-hospital correlation of the observed outcome and accommodates the assumption that underlying differences in quality across hospitals lead to systematic differences in outcomes. While Bayesian methods would be appropriate for these analyses, they are impractical in our extremely large sample sizes. Note also that we use SAS PROC GLIMMIX with default settings to estimate each model. We compared the use of the default fitting algorithm (residual pseudo-likelihood estimation) to use of adaptive quadrature with 12 points and 21 points in the cardiovascular cohort and found no difference in results. Correlation coefficients between different estimation methods = 0.9999, and the maximum difference in RSRR between adaptive quadrature with 21 points and the default glimmix approach is 0.000476, indicating identical rank order. However, adaptive quadrature was much more computationally intensive, requiring several days to run a single model, and could not be performed at all on larger cohorts. Consequently for all analyses, we use the default settings. We model the probability of readmission as a function of patient age, clinically relevant comorbidities, and index condition categories with an intercept for the hospital-specific random effect. We did not test estimation options with rarer outcomes or fewer hospitals, as these were not relevant to our measure.

Specifically, for a given specialty cohort, we estimated a mixed effects logistic regression model as follows. Let $Y_{ij}$ denote the outcome (equal to 1 if patient $i$ is readmitted within 30 days, zero otherwise) for a patient in cohort $C \subseteq \{1, \ldots, 5\}$ at hospital $j$; $Z_{ij}$ denotes a set of risk factors. Let $M$ denote the total number of hospitals and $m_j$ the number of index patient stays in hospital $j$. We assume the outcome is related linearly to the covariates via a logit function with dispersion:

$$\text{logit}(\text{Prob}(Y_{i} = 1)) = \alpha_j + \beta^*Z_{ij} \quad (1)$$

$$\alpha_j = \mu + \omega_j ; \omega_j \sim N(0, \tau^2)$$

where $Z_{ij} = (Z_{i1}, Z_{i2}, \ldots, Z_{ik})$ is a set of $k$ patient-level covariates. $\alpha_j$ represents the hospital specific intercept; $\mu$ is the adjusted average intercept over all hospitals; $\tau^2$ is the between hospital variance component. The mixed effects logistic regression models are estimated using the SAS software system (SAS 9.3 GLIMMIX).

Hospital performance reporting

The previous section describes how the models for each specialty cohort are specified and estimated, using a separate mixed effects logistic regression model for that cohort. Each model is then used to calculate a standardized risk ratio (SRR) for each hospital that contributes index
admissions to that model. These SRRs, weighted by volume, are pooled for each hospital to create a composite hospital-wide SRR.

**SRR for each specialty cohort**

We used the results of each mixed effects logistic regression model to calculate the predicted number of readmissions and the expected number of readmissions at each hospital. The predicted number of readmissions in each cohort was calculated, using the corresponding mixed effects logistic regression model, as the sum of the predicted probability of readmission for each patient, including the hospital-specific (random) effect. The expected number of readmissions in each cohort for each hospital was similarly calculated as the sum of the predicted probability of readmission for each patient, ignoring the hospital specific (random) effect. Using the notation of the previous section, the model-specific risk-standardized readmission ratio is calculated as follows. To calculate the predicted number of admissions pred$_{Cj}$ for index admissions in cohort C=1,...,5 at hospital j, we used

$$\text{pred}_{Cj} = \sum \logit^{-1}(\alpha_j + \beta^*Z_{ij})$$

(2)

where the sum is over all $m_{Cj}$ index admissions in cohort C with index admissions at hospital j. To calculate the expected number exp$_{Cj}$ we used

$$\text{exp}_{Cj} = \sum \logit^{-1}(\mu + \beta^*Z_{ij})$$

(3)

Then, as a measure of excess or reduced readmissions among index admissions in cohort $C$ at hospital $j$, we calculated the standardized risk ratio SRR$_{Cj}$ as

$$\text{SRR}_{Cj} = \frac{\text{pred}_{Cj}}{\text{exp}_{Cj}}$$

(4)

**Risk-standardized hospital-wide 30-day readmission rate**

To report a single readmission score, the separate specialty cohort SRRs were combined into a single value. We created a single score as follows.

For a given hospital, $j$, which has patients in some subset of cohorts $C \subseteq \{1,...,5\}$, calculate the SRR as described for each specialty cohort for which the hospital discharged patients. If the hospital does not have index admissions in a given cohort $c$, then $m_{cj} = 0$ and we take SRR$_{cj} = 1$. Then, calculate the volume-weighted logarithmic mean:

$$\text{SRR}_j = \exp\left( \frac{\sum m_{cj} \log(R_{cj})}{\sum m_{cj}} \right)$$

(5)

where the sums are over all specialty cohorts; note that if a hospital does not have index admissions in a given cohort ($m_{cj} = 0$) that cohort contributes nothing to the overall score SRR$_j$. This value, SRR$_j$, is the hospital-wide SRR for hospital $j$. To aid interpretation, this ratio is then multiplied by the overall national raw readmission rate for all index admissions in all cohorts, $\overline{Y}$, to produce the RSRR$_j$.

$$\text{RSRR}_j = \text{SRR}_j \times \overline{Y}$$

(6)
Creating Interval Estimates

Because the statistic described in Equation 6, that is, \( RSRR_j \), is a complex function of parameter estimates, we use the re-sampling technique, bootstrapping, to derive an interval estimate. (See Normand SL, Wang Y, Krumholz HM. Assessing surrogacy of data sources for institutional comparisons. Health Serv Outcomes Res Method. 2007;7(1-2):79-96.) Bootstrapping has the advantage of avoiding unnecessary distributional assumptions.

Algorithm:

Let \( M \) denote the total number of hospitals in the sample. We repeat steps 1–4 below for \( b = 1, 2, \ldots, B \) times:

1. Sample \( M \) hospitals with replacement.
2. Fit the five cohort mixed effects logistic regression models using all patients within each sampled hospital. As starting values, we use the parameter estimates obtained by fitting the model to all hospitals. If some hospitals are selected more than once in a bootstrapped sample, we treat them as distinct so that we have \( M \) random effects to estimate the variance components. At the conclusion of Step 2, we have
   a. \( \beta^{(b)} \), the vector of coefficients, and the corresponding variance covariance matrix \( V \).
   b. \( \mu^{(b)} \), the average hospital rate; \( \tau^{2(b)} \), the between hospital variance, and
   c. the set of hospital-specific intercepts and corresponding variances; \( \{\alpha_j^{(b)}\} \), \( \text{var}[\alpha_j^{(b)}] : j = 1, 2, \ldots, M \}
3. We generate a hospital random effect by sampling from the distribution of the hospital-specific distribution obtained in Step 2c. We approximate the distribution for each random effect by a normal distribution. Thus, we draw \( \alpha_j^{(b^*)} \sim N(\alpha_j^{(b)}, \text{var}[\alpha_j^{(b)}]) \) for the unique set of hospitals sampled in Step 1.
4. Within each unique hospital \( j \) sampled in Step 1, and using index admissions \( i=1, \ldots, m_j \) in that hospital, we calculate \( SRR^*_j \) and then \( RSRR^*_j \) as in equations (5) and (6).

Ninety-five percent interval estimates (or alternative interval estimates) for the hospital-standardized outcome can be computed by identifying the 2.5th and 97.5th percentiles of randomly half of the \( B \) estimates (or the percentiles corresponding to the alternative desired intervals).